

1.2b The Factor Theorem

Note Title

11/09/2012

Determine the remainder when $3x^4 + 7x^3 - x^2 + 14x - 3$ is divided by

a) $x - 1$

$$3(1)^4 + 7(1)^3 - (1)^2 + 14(1) - 3 = 20$$

(NOT A FACTOR)

b) $x + 3$

$$3(-3)^4 + 7(-3)^3 - (-3)^2 + 14(-3) - 3 = 0$$

(IS A FACTOR)

The factor theorem:

If $P(a) = 0$ then $x - a$ is a factor.

$$\begin{array}{c} 54 \\ \wedge \\ 2 \quad 27 \\ \wedge \\ 3 \quad 9 \\ \wedge \\ 3 \quad 3 \end{array}$$

Once we have a factor we can divide it out to find other factors.

Factor fully.

$$2x^3 - 3x^2 - 3x + 2$$

$$2(-1)^3 - 3(-1)^2 - 3(-1) + 2 = 0$$

So $x + 1$ is a factor

Try factors of 2
($\pm 1, \pm 2$)

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -3 & 2 \\ & & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$
$$2x^2 - 5x + 2$$

$$\text{So } 2x^3 - 3x^2 - 3x + 2 = (x+1)(2x^2 - 5x + 2)$$

Now factor $2x^2 - 5x + 2$

$$\begin{aligned} &2x^2 - 4x - x + 2 \\ &2x(x-2) - 1(x-2) \\ &(x-2)(2x-1) \end{aligned}$$

$$\text{So } 2x^3 - 3x^2 - 3x + 2 = (x+1)(x-2)(2x-1)$$

* Can check by expanding ☺

Factor fully.

$$2x^3 + 3x^2 - 11x - 6$$

Try $\pm 1, \pm 2, \pm 3, \pm 6$