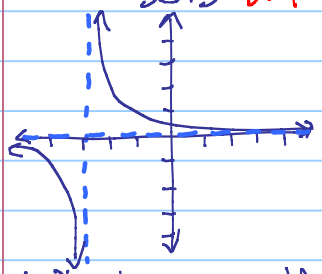
2.3a Analyzing Rational Functions

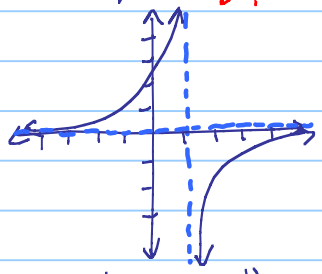
Note Title 26/09/2012

Draw each graph on a separate grid.

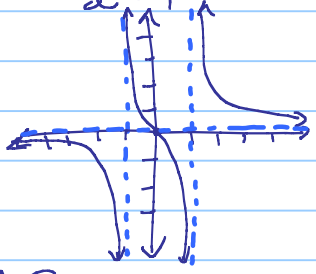
1) $y = \frac{2}{x+3}$ D: 1



2) $y = \frac{3}{1-x}$ D: 1



3) $y = \frac{x}{x^2-1}$ D: 2



What are the asymptotes of each?

1) $x = -3$, $y = 0$

2) $x = 1$

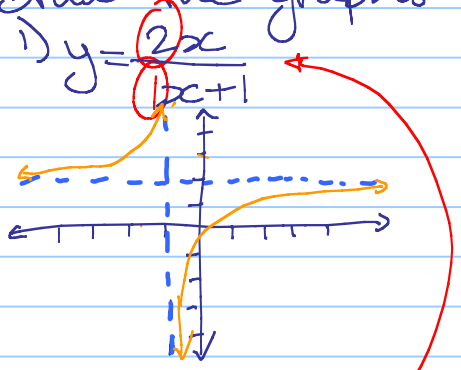
3) $x = \pm 1$

* If the degree of the denominator is greater, we have a horizontal asymptote of $y = 0$.

$$\lim_{x\to\infty} \frac{3\cdot\frac{1}{x}}{(1-x)\frac{1}{x}} = \frac{0}{0-1} = 0$$

Draw the graphs of

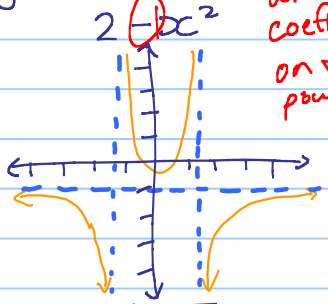
1) $y = \frac{2x}{2x+1}$



Vertical: $x = -1$

Horizontal: $y = 2$

2) $y = \frac{x^2-1}{2-x^2}$ can just look at the coefficients on the highest powers of x



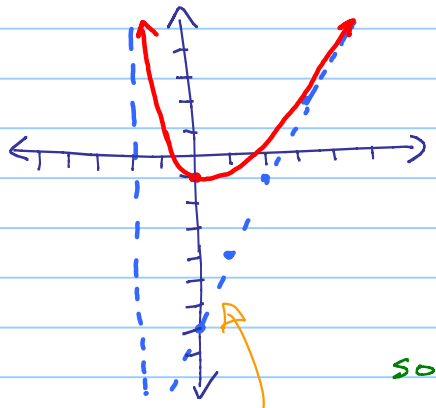
$x = \pm\sqrt{2}$

$y = -1$

$$\lim_{x\to\infty} \frac{(x^2-1)\frac{1}{x^2}}{(2-x^2)\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{\frac{2}{x^2}-1} = \frac{1}{-1} = -1$$

* When the degrees are the same, there will be a horizontal asymptote of $y = k$, $k \neq 0$.

Draw the graph of $y = \frac{3x^2 - x - 2}{x+2}$



slant or oblique asymptote of $y = 3x - 7$

Divide & write the division statement $\frac{P}{D} = Q + \frac{R}{D}$

$$\begin{array}{r|rrr} -2 & 3 & -1 & -2 \\ & & -6 & 14 \\ \hline & 3 & -7 & \boxed{12} \end{array}$$

we have a vert. asymptote

so $\frac{3x^2 - x - 2}{x+2} = 3x - 7 + \frac{12}{x+2}$

as $x \to \infty$, $y = 3x - 7 + \frac{12}{x+2} \to 0$

$y \approx 3x - 7$

* If the degree of the numerator is greater, either
  a) the bottom is a factor and the graph will have a hole OR
  b) the graph will have a vertical asymptote and (possibly) a slant asymptote.