3.5 Inverse Functions

Note Title 09/10/2012

An inverse function will undo what the original function does.

Original: $y=f(x)$ $\qquad f(\text{input}) = \text{output}$

inverse: $x=f(y)$ $\qquad f(\text{output}) = \text{input}$

$y=f^{-1}(x)$

Not an exponent!

$f^{-1}(x) \neq \dfrac{1}{f(x)}$

$\boxed{\text{Switches } x \text{ \& } y}$

Find the inverse of the function $f(x)=2x-1$

$f(3) = 2(3)-1$
$= 6-1$
$= 5$

Inverse: $f^{-1}(x) = \dfrac{x+1}{2}$

Check: $f^{-1}(5) = \dfrac{5+1}{2}$
$= \dfrac{6}{2}$
$= 3$ ☺

Algebraically:

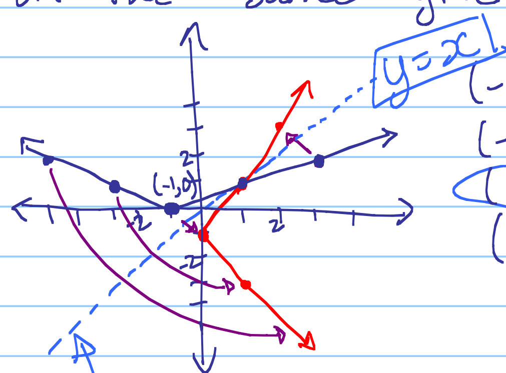
$y = 2x-1$

Inverse: $x = 2y-1$

Solve for y: $x+1 = 2y$

$\dfrac{x+1}{2} = y$

For the graph of $y=f(x)$ shown, draw $y=f^{-1}(x)$ on the same grid.



$(-1,0) \rightarrow (0,-1)$

$(-3,1) \rightarrow (1,-3)$

$\boxed{(1,1) \rightarrow (1,1)}$

$(3,2) \rightarrow (2,3)$

If $y=x$ you have an invariant point

Reflection in the line $y=x$

Find the inverse of $y = x^2 + 1$ and draw both graphs.

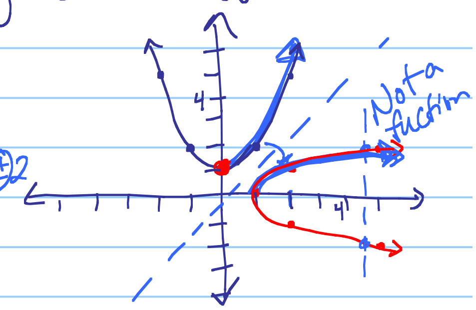
Inverse: $x = y^2 + 1$

$x - 1 = y^2$

$y = \pm\sqrt{x-1}$

$y^2 = 4$

$y = \pm 2$



Is the inverse a function? No

If we restrict the domain of $y = f(x)$ to $x \geq 0$ then the inverse would be a function.

(OR $x \leq 0$ would work also.)

Example: Find the inverse of $f(x) = (x-2)^2 - 1$ and the domain restriction needed to make the inverse a function.

x value of vertex

$x = (y-2)^2 - 1$

$x + 1 = (y-2)^2$

$\pm\sqrt{x+1} = y - 2$

$y = \pm\sqrt{x+1} + 2$

Domain restriction: $x \geq 2$

(OR $x \leq 2$)

Find the domain & range of $y = \dfrac{1}{x+2}$

D: $x \neq -2$

R: $y \neq 0$

Inverse: $x = \dfrac{1}{y+2}$

$x(y+2) = 1$

$y + 2 = \dfrac{1}{x}$

$y = \frac{1}{x} - 2$ D: $x \neq 0$