5.5 Laws of Logarithms

Note Title 05/11/2012

Evaluate.

a) $\log_2 2^2$ b) $\log_2 2^3$ c) $\log_2 2^2 \cdot 2^3$

$= 2$ $= 3$ $= 5$

* Same as $\log_2 2^2 + \log_2 2^3$

Product Rule: $\log_b (x \cdot y) = \log_b x + \log_b y$

(power; exponents)

Proof: Let $\log_b x = m$ and $\log_b y = n$

$b^m = x$ $b^n = y$

$\log_b x \cdot y = \log_b b^m \cdot b^n$

$= \log_b b^{m+n}$

$= m+n$

$\therefore \log_b x \cdot y = \log_b x + \log_b y$

Try:

1) Evaluate $\log 2 + \log 50 = \log 2 \cdot 50$

$= \log 100$

$= 2$

(no base means base 10)

in calc: $\log 2 = 0.3010\ldots$

$\log 50 = 1.6989\ldots$

$\log 2 + \log 50 = 2$

Quotient Rule: $\log_b \frac{x}{y} = \log_b x - \log_b y$

* Try proving!

Try: 1) Evaluate $\log_3 54 - \log_3 6$

$= \log_3 \frac{54}{6}$

$= \log_3 9 = 2$

2) Given $\log_2 5 = a$, write an expression for $\log_2\left(\frac{64}{25}\right)$.

$= \log_2 64 - \log_2 25$

$= 6 - \log_2 25$

$= 6 - 2a$

$\log_2 5 = a$

$(2^a)^2 = (5)^2$

$2^{2a} = 25$

$\log_2 25 = 2a$

Power Law: $\log_b x^{\boxed{k}} = k \cdot \log_b x$

Try: 1) Write as a single logarithm:

$\frac{1}{2}\log A - 2\log B + 3\log C$

$= \log A^{1/2} - \log B^2 + \log C^3$

$= \log \frac{A^{1/2}}{B^2} + \log C^3$

$= \log \frac{A^{1/2} C^3}{B^2}$

2) Evaluate $3\log_2 3 - 2\log_2 18 + \log_2 3$

$\log_2 3^3 - \log_2 18^2 + \log_2 3$

$= \log_2 \frac{3^3 \cdot 3}{18^2}$

$= \log_2 \frac{3^4}{2^2 \cdot 9^2}$

$= \log_2 \frac{81}{2^2 \cdot 81}$ OR $\log_2 \frac{3^4}{2^2 \cdot 3^4}$

$= \log_2 \frac{1}{2^2}$

$= \log_2 2^{-2}$

$= -2$