5.6a Change of Base

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How do we use a calculator to evaluate $\log_2 5$?

Let $\log_2 5 = x$

$2^x = 5$

so $\log 2^x = \log 5$

$\Rightarrow \dfrac{x\cancel{\log 2}}{\cancel{\log 2}} = \dfrac{\log 5}{\log 2}$

$\therefore \log_2 5 = \dfrac{\log 5}{\log 2} = 2.3$

Evaluate $\log_3 29$ to 3 decimal places.

$= \dfrac{\log 29}{\log 3} = 3.065$

In general: Let $\log_b a = x$

$b^x = a$

so $\log_c b^x = \log_c a$

$\dfrac{x\cancel{\log_c b}}{\cancel{\log_c b}} = \dfrac{\log_c a}{\log_c b}$

$\therefore \boxed{\log_b a = \dfrac{\log_c a}{\log_c b}}$

Write $\log_2 5$ as a base 3 logarithm.

$= \dfrac{\log_3 5}{\log_3 2}$

$\log_3\left(\frac{5}{2}\right) = \log_3 5 - \log_3 2$

Write $\dfrac{\log_5 x}{\log_5 y}$ as a single logarithm.

$$\overline{\log_5 y}$$

$$= \log_y x$$

Write $\log_{\frac{1}{2}} x$ as a base 2 logarithm.

$$\log_{\frac{1}{2}} x = \frac{\log_2 x}{\log_2\left(\frac{1}{2}\right)}$$

$$= \frac{\log_2 x}{-1}$$

$$= -\log_2 x$$

$$= \log_2 x^{-1}$$

$$\log_{\frac{1}{2}} x = \log_2\left(\frac{1}{x}\right)$$

When the new base is a power of the original:

$$\log_b x = \frac{n \log x}{n \log b}$$

$$= \frac{\log(x^n)}{\log(b^n)}$$

$$= \log_{b^n} x^n$$

$$\therefore \log_b x = \log_{b^n} x^n$$

Given $\boxed{\log_6 x} = \boxed{6}$, what is

a) $\log_{(1/6)^{-1}} (x)^{-1}$ b) $\log_{36} x$ c) $\log_{(\sqrt{6})^2} x^2$

$= \cancel{\log_6\left(\frac{1}{x}\right)}$ $= \log_{6^2} x$ $= \log_6 x^2$

$= \log_6 x^{-1}$

$= -\boxed{\log_6 x}$

$= -6$

$= \log_{(6^2)^{1/2}} x^{1/2}$

$= \log_6 x^{1/2}$

$= \frac{1}{2}(6)$

$= 3$

$= 2(6)$

$= 12$

Evaluate $\log_{16} 2 = \frac{1}{4}$

A) $\log_{16} 2 = \log_{2^4} 2$

$= \log_2 2^{1/4}$

$= \frac{1}{4}$

B) $\log_{16} 2 = \dfrac{\log_2 2}{\log_2 16}$

$= \dfrac{1}{4}$

C) $\log_{16} 2 = x$

$16^x = 2$

$2^{4x} = 2^1$

$4x = 1$

$x = \frac{1}{4}$