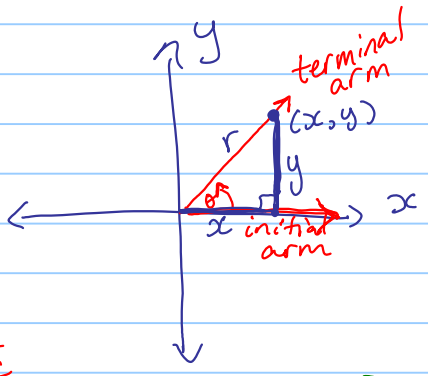
6.1 Trig Ratios

Note Title 16/11/2012

By drawing an angle on a coordinate plane, we can relate the trigonometric ratios to x and y.

Standard Position:
- vertex at origin (0,0)
- initial arm on positive x-axis
- positive angle is ccw



$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

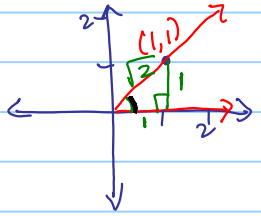
$$\csc\theta = \frac{r}{y} \quad \sec\theta = \frac{r}{x} \quad \cot\theta = \frac{x}{y}$$

$\csc\theta$ ↑ $\frac{1}{\sin\theta}$; $\sec\theta$ ↑ $\frac{1}{\cos\theta}$; $\cot\theta$ ↑ $\frac{1}{\tan\theta}$

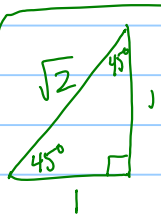
$$r^2 = x^2 + y^2$$

$$\boxed{r = \sqrt{x^2+y^2}}$$

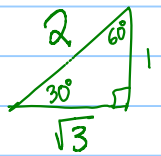
If the point $(1,1)$ lies on the terminal arm, what are the values of all six trig ratios of θ? What is the value of θ?



$r = \sqrt{1^2+1^2} = \sqrt{2}$



45-45-90 triangle



30-60-90 triangle

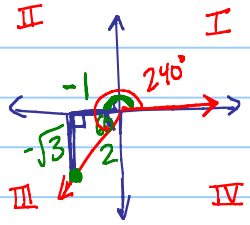
$\sin\theta = \frac{1}{\sqrt{2}}$ $\quad \csc\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \sec\theta = \sqrt{2}$

$\tan\theta = 1$ $\quad \cot\theta = 1$

$\theta = 45°$

What is the exact value of $\sin(240°)$?

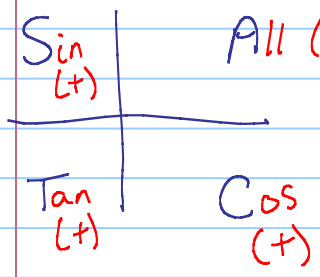


reference angle is between the terminal arm and the closest x-axis.

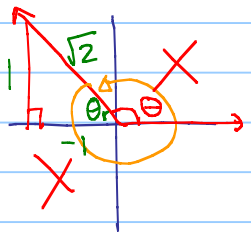
$\theta_r = 240° - 180° = 60°$

So $\sin(240°) = -\frac{\sqrt{3}}{2}$

*Can also use CAST to figure out +/−



Given that $\sec\theta = -\sqrt{2}$ (cos (−)) and $\tan\theta < 0$ (tan (−)), find the other trig ratios (exact values) and θ.



$\sec\theta = -\sqrt{2}$

$\frac{1}{\cos\theta} = \frac{-\sqrt{2}}{1}$

$\therefore \cos\theta = \frac{-1}{\sqrt{2}}$ (A, H)

$\sin\theta = \frac{1}{\sqrt{2}}$ $\quad \tan\theta = -1$

$\csc\theta = \sqrt{2}$ $\quad \cot\theta = -1$

$\theta_r = 45°$

$\theta = 180° - 45°$

$= 135°$

If $0 \le \theta < 360°$

Without this restriction, we could have $\theta = 135° + 360$

$= 495°$

OR $135° - 360$

$= -225°$

These are called coterminal angles

General Solution:

$\theta = 135° + 360°n$

$n \in \mathbb{Z}$ (integers)