

7.3 Reciprocal & Quotient Identities

Note Title

10/12/2012

An identity is an equation that is true for all values of the variable (except restrictions).

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

- Graphs overlap
- Same value for any value of the variable

Verifying:

- Check
- 2 methods:
 - See if graphs overlap
 - Plug in a value

Proof:

- Algebraically

Is $\sin \theta \sec \theta \cot \theta = 1$ an identity?

$$\text{Graph } y = \sin x \sec x \cot x = \frac{\sin x}{\cos x \tan x}$$

Looks like $y=1$ ✓

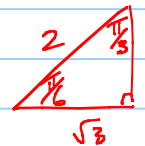
Plug in $x = \frac{\pi}{6}$:

$$\sin \frac{\pi}{6} \sec \frac{\pi}{6} \cot \frac{\pi}{6} \quad | \quad 1$$

$$\left(\frac{1}{2}\right) \left(\frac{2}{\sqrt{3}}\right) \left(\frac{\sqrt{3}}{1}\right)$$

$$1 \neq 1$$

LHS \neq RHS



Algebraic proof:

$$\frac{\sin \theta \sec \theta \cot \theta}{\sin \theta \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}} \quad | \quad 1$$
$$1 \neq 1$$

Restrictions:
 $\cos \theta \neq 0, \sin \theta \neq 0$

Solving Equations using Identities:

Solve $3\cos x + 1 = 2\sec x$ for $0 \leq x < 2\pi$.

$$\cos x (3\cos x + 1) = \frac{2}{\cos x} \quad \cos x \neq 0$$

$$3\cos^2 x + \cos x = 2$$

$$3\cos^2 x + \cos x - 2 = 0 \quad \text{Let } a = \cos x$$

$$3a^2 + a - 2 = 0$$

$$3a^2 + 3a - 2a - 2 = 0$$

$$3a(a+1) - 2(a+1) = 0$$

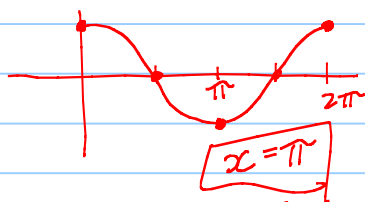
$$(a+1)(3a-2) = 0$$

$$a = -1, \frac{2}{3}$$

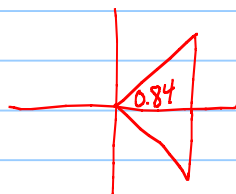
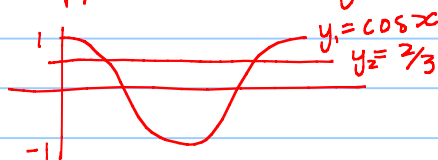
$$\cos x = -1, \frac{2}{3}$$

$$\cos x = -1$$

$$\cos x = \frac{2}{3}$$



Approximate using calc:



$$\theta = \cos^{-1}\left(\frac{2}{3}\right) = 0.84$$

$$x = 0.84, 2\pi - 0.84$$

$$x = 0.84, 5.44$$

So $x = 0.84, \pi, 5.44$