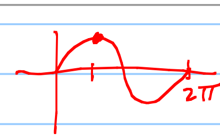
7.5 Sum & Difference Identities

Note Title 12/12/2012

Is $\sin(A+B) = \sin A + \sin B$?



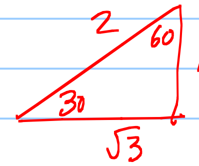
Try $A = 30^\circ$ $B = 60^\circ$

$\sin(30+60)$ | $\sin 30 + \sin 60$

$= \sin 90$ | $\frac{1}{2} + \frac{\sqrt{3}}{2}$

$= 1$

$\neq \; = \frac{1+\sqrt{3}}{2}$



$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$\sin(30+60)$ | $\sin 30 \cos 60 + \cos 30 \sin 60$

$= \sin 90$ | $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= 1$ | $= \frac{1}{4} + \frac{3}{4}$

$= 1$

Sum & Difference Identities:

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

Find the exact value of $\sin \frac{\pi}{12}$.

We know: $\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}$

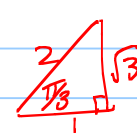
$\frac{4\pi}{12}, \frac{3\pi}{12}, \frac{2\pi}{12}, \frac{6\pi}{12}$

$\sin\frac{\pi}{12} = \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \sin\left(\overset{A}{\frac{\pi}{3}} - \overset{B}{\frac{\pi}{4}}\right)$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$



$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

Prove $\cos\left(\overset{A}{\frac{\pi}{2}} - \overset{B}{x}\right) = \sin x$

$\cancel{\cos\frac{\pi}{2}} \cos x + \cancel{\sin\frac{\pi}{2}} \sin x$

$(0) \cos x + (1) \sin x$

$= \sin x$

QED

LHS = RHS