Note Title 14/11/2012

Continuous Growth & Future Value

If \$1 is compounded n times in a year at a rate of 100%, the value would be

$$\left(1+\frac{1}{n}\right)^n$$

Continuously: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.718281828\ldots$

called $\underline{e}$

this in the <u>natural</u> base

For continuous growth: $A = A_0 e^{rt}$ ← time

amount

initial amount

growth rate

Example:

A population of 2.3 million is growing <u>continuously</u> at a rate of 8% per year. What will the population be in 5 years?

Approximate using yearly growth:

$$A = 2.3(1.08)^5$$
$$= 3.38 \text{ million}$$

Exact value using continuous growth:

$$A = 2.3e^{(0.08(5))}$$
$$= 3.43 \text{ million}$$

How long will it take to reach 5 million?

$$\frac{5}{2.3} = \frac{\cancel{2.3}e^{0.08t}}{\cancel{2.3}}$$

$$\ln\left(\frac{5}{2.3}\right) = \ln e^{\underline{0.08t}}$$

$\log_e$ OR $\ln$

"lawn"

is the natural logarithm

$$\log_2 2^5 = 5$$

$$\frac{\ln\left(\frac{5}{2.3}\right)}{0.08} = \frac{\cancel{0.08}t}{\cancel{0.08}}$$

$$\log_e e^{0.08t} = 0.08t$$

$$t = 9.7 \text{ years}$$

Bacteria doubling every 15 minutes

Approximate:

$$1(2)^{4\times24} = 7.9\times10^{28}$$

Exact (continuous)

$$1e^{1\times96} = 4.9\times10^{41}$$

Future Value:

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

n = # of monthly investments

R = amount of regular investment

i = interest rate

If you invest your \$400/month at 8% compounded monthly, how much would you have in 40 years?

$$FV = \frac{400\left[\left(1+\frac{0.08}{12}\right)^{480} - 1\right]}{\left(\frac{0.08}{12}\right)}$$

$$= \$1,396,403.13$$

How much interest is this?

Money invested: $\$400 \times 12 \times 40 = \$192,000$

$\$1,396,403.13 - \$192,000$

$= \$1,204,403.13$

$\approx \$1.2$ million in interest